Simulation of Gravitational Lensing

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Abstract

Dark matter has been one of the most thoroughly studied areas of cosmology. One of the most useful tools at the disposal of astrophysicists in studying dark matter in galaxies and galaxy clusters is Gravitational Lensing. Through the study of gravitational deflection of the light of background objects due to foreground (e.g. galactic) deflectors, one can obtain a two dimensional mass density projection of the unseen matter comprising most of the mass of the universe. Ellipticity changes in the background objects provide a method of quantifying distortions of such objects. Shape changes, convergence and shear, provide a means to relate ellipticity to mass density. Two programs have been developed to simulate light ray deflection and the shearing effects of gravitational lensing. Such software will be used in further study to relate shear to mass density, with particular interest in the commonly accepted mass density model, the Navarro, Frenk and White (NFW) model.

Introduction

Since the 1930's, the discrepancy between luminous mass measurements of galaxy clusters and their dynamics, such as velocity dispersion, has been all too clear. Such a disagreement poses a problem in the understanding of gravitation and the overall structure and evolution of the universe. The commonly accepted solution to this problem, namely dark matter, suggests that the amount of mass that is visible does not represent the entire amount of mass present in the universe. Should this be the case, one question arises: how can one study something that cannot be seen? Gravitational lensing addresses this problem.

One of the early successes of Einstein's new theory of gravity was the prediction of light deflection by massive objects. Light rays passing near a massive object, such as a star, are deflected from their paths as a result of the curving of space and time. Such a deflection is illustrated in Figure 1. In this example, a light ray is being traced from the source, S, to the observer plane at the bottom of Figure 1. The following simulations are preformed using the thin lens approximation, valid for cosmological studies where the deflection occurs over distance scales much less than the overall line-of-sight distances, i.e. D_d or D_{ds} . In such an approximation, the lens can be treated as a two dimensional deflector within the plane shown in Figure 1. Without the middle deflecting plane, a light ray would follow the straight path, making an angle, β , with the perpendicular line through all three planes. However, with a large enough mass between the source and observer planes, the light ray is instead deflected from its path by an angle, α . For an extended background source, this light ray deflection results in shape distortion, including displacement and parity inversion. A few examples are given in Figure 2.

In Figure 2, two examples of distortion of extended sources have been simulated. In both examples, an extremely large mass has been placed directly in between the background source (left panel) and the observer, who sees the distorted image (right panel). The centers of each of the three planes, observer, source and deflecting mass, are collinear. In the top panels, the background image, a circle, is slightly offset from the center of the plane. In the case of strong lensing, there are two images of the background source, a larger image on the same side of the center point as the source and a much smaller image on the opposite side of the center. If the original source was more than a solid circle, one would also see that the larger image is inverted. This change in polarity is (slightly) more readily visible in the bottom panels. As the circle in the top left panel is moved into alignment with the lens and observer, the two lensed images on the right will merge together in a so called Einstein ring. On the other hand, as the background source is moved out of alignment, there is a point at which the lensed image will instead resemble the original background image, as it should when the source is sufficiently removed from the lensing system.

Lensing Simulation

When simulating gravitational lensing, two techniques are commonly used. First, one can directly calculate the deflection angle of each light ray. Once the deflection angles are known, the light ray can be traced backwards to its apparent origin in the source plane. This technique is known as Ray Tracing. Figure 2 shows two examples of this method of simulation. In ray tracing, the deflection angle is calculated by performing a vector sum of the deflection angles over all of the mass in the foreground deflecting mass. For this reason, the calculation is quite time consuming. To save on time, the simulations here were performed with the use of volunteer distributive computing, allowing for the use of parallel computing[1].

Another method of gravitational lensing simulation is calculating the shape distortion of background objects. Background objects experience two types of distortion, magnification (convergence) and stretching (shear)[2]. Convergence provides direct link to the mass density of the deflecting, foreground mass. In fact, in standard gravitational theory, the convergence is directly proportional to the mass density. In non-standard theories, convergence is still related to mass, though not in such a linear manner[3]. Both convergence and shear are related to the gravitational potential of the deflecting mass[4]. Therefore, if one can determine the amount of shear a background image experiences, a direct measurement of the foreground mass can be obtained, producing a two-dimensional mass density of the foreground mass. It is worth noting that this effect is independent of the type of matter in the foreground; it is only required that the matter couple to the gravitational field. Thus, both luminous and dark matter are “observed” via gravitational lensing, making lensing a powerful tool in studying dark matter.

Shear Map

As stated earlier, shear and convergence can be related to each other through the gravitational potential, for example through Fourier transforms. With the use of a computer, this can most efficiently be done with the use of Fast Fourier Transform (FFT). In fact, on an Intel Celeron 2.53GHz PC with 1.25GB RAM running Fedora Core 7 Linux the above mentioned ray trace simulation takes roughly eight hours. On the same computer, the FFT based shear program takes less than five minutes!

In weak lensing, i.e. when a source is outside the region where multiple images occur, for example by galaxy clusters, one does not see such effects as shown in Figure 2. In fact, the amount of shear in weak lensing is so small that shear is only revealed through statistical analysis. The main effect of lensing is a change in ellipticity, beyond the intrinsic ellipticity, so as to align the approximately elliptical image tangentially to a circle centered on the lens, corresponding to the peak in the mass density of the distorting mass[5]. Figure 3 shows this weak lensing effect. Since no particular direction is singled out within the galaxy cluster, galaxy orientation should be random. However, after lensing is taken into account, the orientation tends to be tangentially aligned, as shown in Figure 3.

Usefulness of Shear Map

Once the shear map has been obtained, a two-dimensional mass density profile can be created. Using the shear map shown in Figure 3, Hoekstra[5] et al 1998 were able to deduce the mass density map of the foreground, lensing galaxy cluster, shown in Figure 4. Another example of the use of this technique is in the famous Bullet Cluster. In what was deemed a proof of the existence of Dark Matter, gravitational lensing was used to show that peaks in the mass density are well off-set from the visible mass, ie x-ray emitting plasma[3].

One of the current goals of cosmology is the creation of a general model the shape of dark matter in galaxy clusters. Various models have been proposed. In studying dark matter, it would be convenient to have one analytical method of calculating shear based on a specific mass model, such as the NFW model[6]. In such work, it will be necessary to have simulations to validate theoretical work. The shear and ray trace programs will serve this function.

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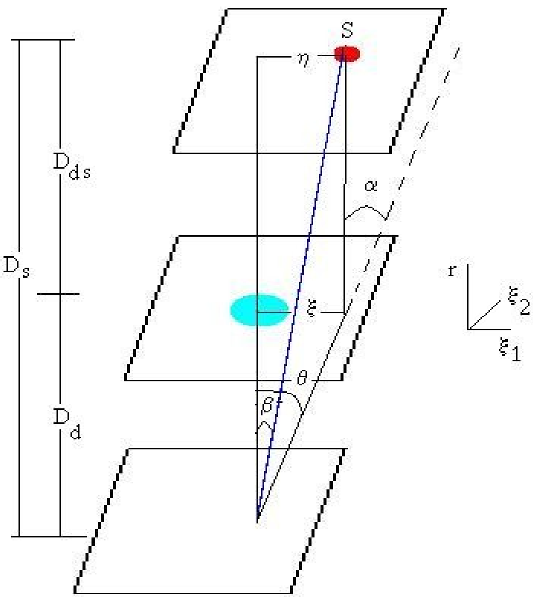

Figure 1: Light deflection of a background object by an angle, α . The dotted line shows the apparent position of the background image, S. The light ray follows a curved path due to the foreground mass. The unlensed path is show to illustrate the deviation of the light ray.

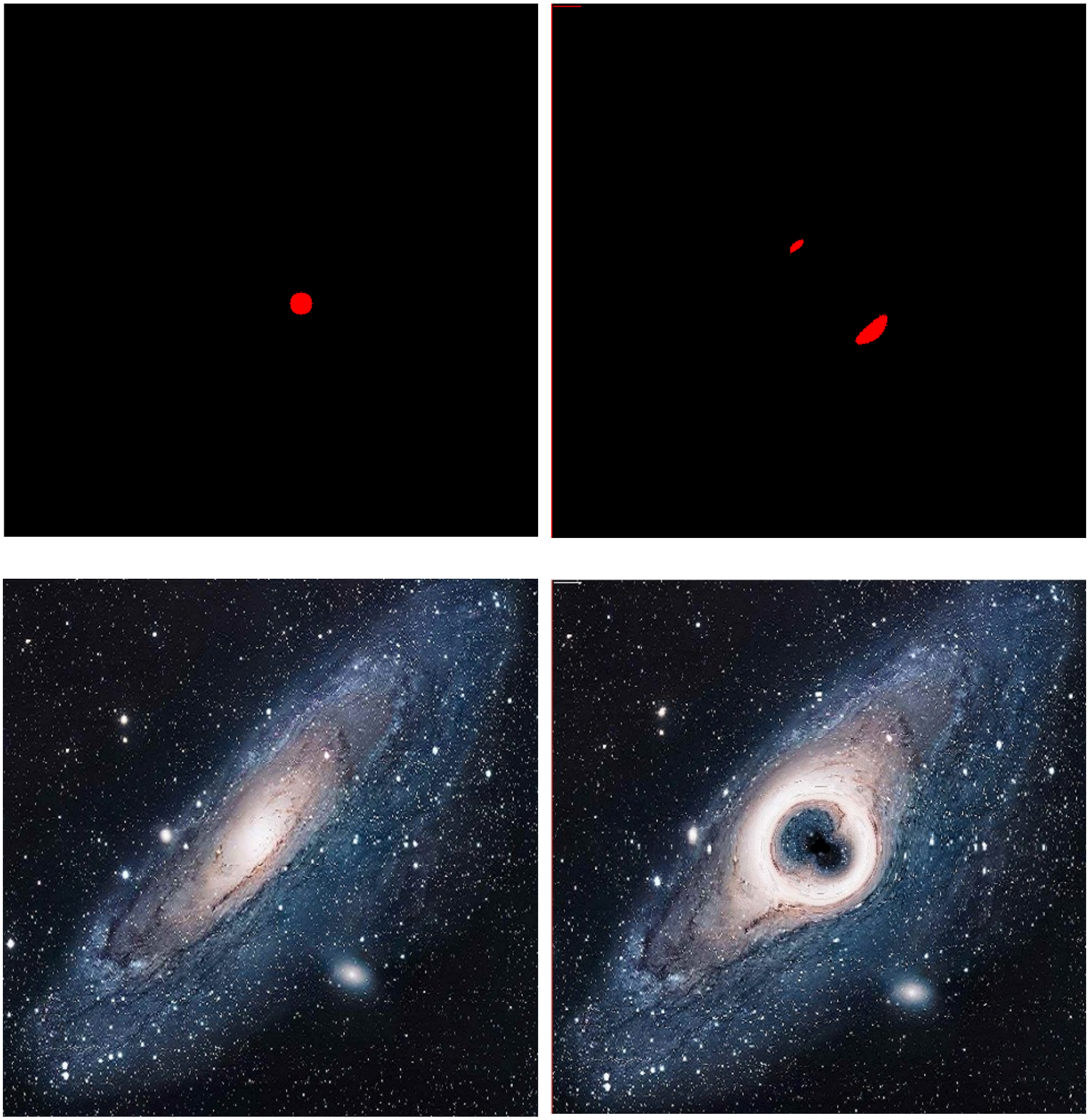

Figure 2: Examples of Gravitational Light Deflection. Each Original Image (left) has been deformed by a large mass placed between it and the observer (right).

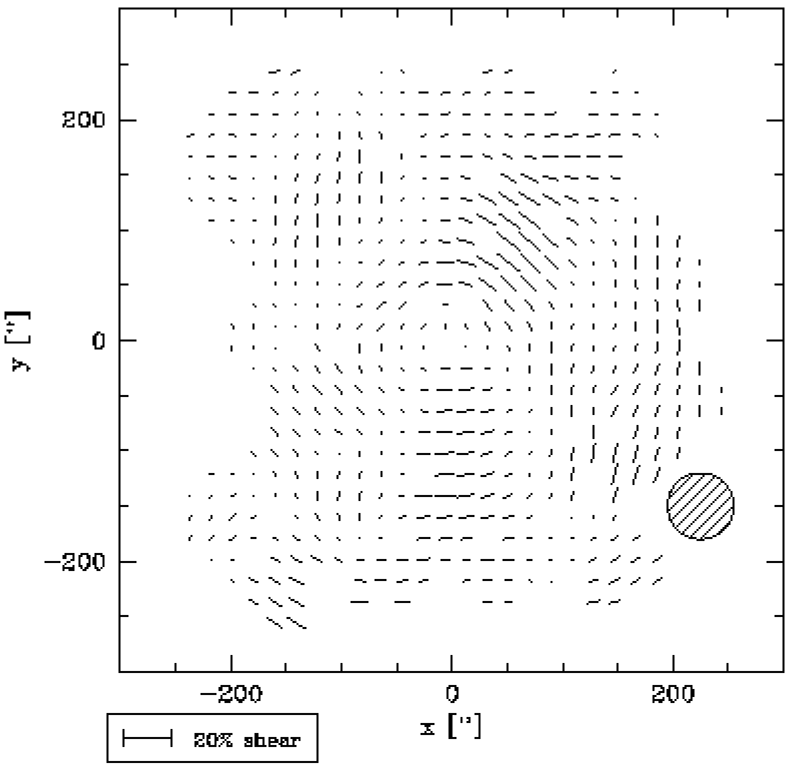

Figure 3: Shear field of the galaxy cluster Cl 1358+62. Line lengths represent the amount of distortion. The tangential shear is a common property of weak lensing. Horizontal and Vertical distances are measured in arcseconds (1" = 4.23 kpc in the Concordance Cosmology at the redshift of z=0.33). Figure taken from Hoekstra[5] et al 1998

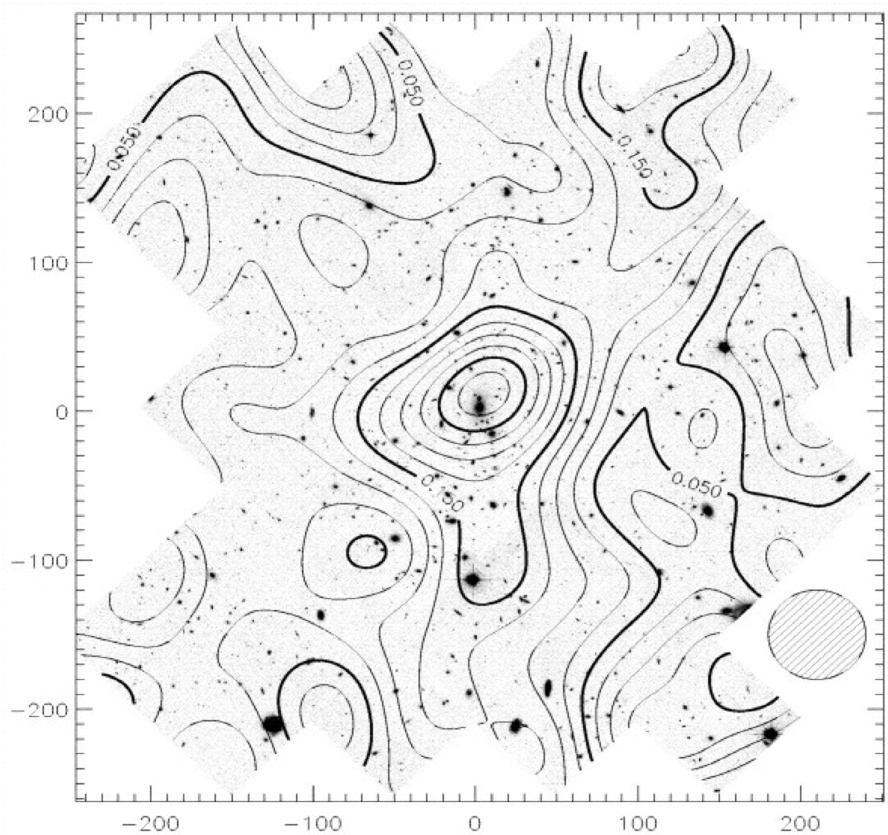

Figure 4: Mass density profile created by Hoekstra[5] et al 1998 using the shear map shown in Figure 3. The mass density is represent using contour mapping. The mass center corresponds well to the lensing center in Figure 3. Again, horizontal and vertical distances are measured in arcseconds (1” = 4.23 kpc, cf Figure 3).

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